9.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. An ________ ________ is a function whose domain is the set of positive integers.
2. A sequence is a ________ sequence when the domain of the function consists only of the first n positive integers.
3. When you are given one or more of the first few terms of a sequence, and all other terms of the sequence are defined using previous terms, the sequence is defined ________.
4. If n is a positive integer, then n ________ is defined as $n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots (n-1) \cdot n$.
5. For the sum $\sum_{i=1}^{n} a_i$, i is the ________ of summation, n is the ________ limit of summation, and 1 is the ________ limit of summation.
6. The sum of the terms of a finite or infinite sequence is called a ________.

Skills and Applications

Writing the Terms of a Sequence

In Exercises 7–22, write the first five terms of the sequence. (Assume that n begins with 1.)

7. $a_n = 4n - 7$
8. $a_n = -2n + 8$
9. $a_n = (-1)^{n+1} + 4$
10. $a_n = 1 - (-1)^n$
11. $a_n = (-2)^n$
12. $a_n = \left(\frac{1}{2}\right)^n$
13. $a_n = \frac{2}{3}$
14. $a_n = 6(-1)^{n+1}$
15. $a_n = \frac{1}{3}n^3$
16. $a_n = \frac{1}{n^2}$
17. $a_n = \frac{n}{n+2}$
18. $a_n = \frac{6n}{3n^2 - 1}$
19. $a_n = n(n-1)(n-2)$
20. $a_n = n(n^2 - 6)$
21. $a_n = (-1)^n\left(\frac{n}{n+1}\right)$
22. $a_n = \frac{(-1)^{n+1}}{n^2 + 1}$

Finding a Term of a Sequence

In Exercises 23–26, find the missing term of the sequence.

23. $a_n = (-1)^n(3n - 2)$, $a_{25} = \square$
24. $a_n = (-1)^{n-1}[n(n-1)]$, $a_{16} = \square$
25. $a_n = \frac{4n}{2n^2 - 3}$, $a_{11} = \square$
26. $a_n = \frac{4n^2 - n + 3}{n(n-1)(n+2)}$, $a_{13} = \square$

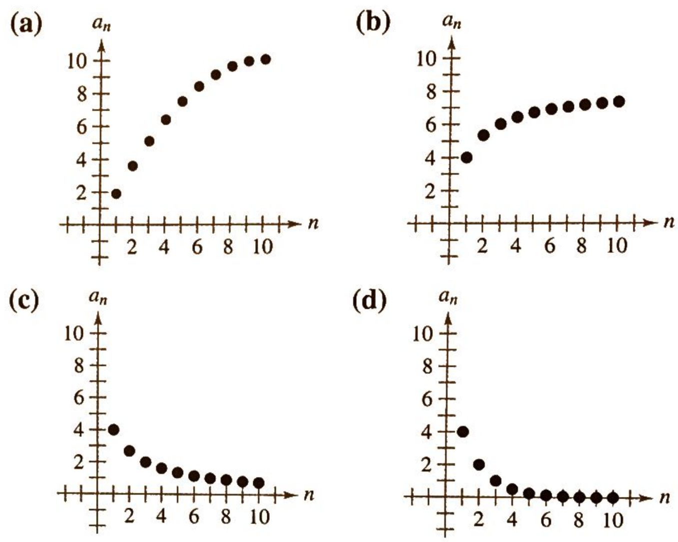
Graphing the Terms of a Sequence

In Exercises 27–32, use a graphing utility to graph the first 10 terms of the sequence. (Assume that n begins with 1.)

27. $a_n = \frac{2}{3}n$
28. $a_n = 3n + 3(-1)^n$
29. $a_n = 16(-0.5)^{n-1}$
30. $a_n = 8(0.75)^{n-1}$
31. $a_n = \frac{2n}{n+1}$
32. $a_n = \frac{3n^2}{n^2 + 1}$

Matching a Sequence with a Graph

In Exercises 33–36, match the sequence with the graph of its first 10 terms. [The graphs are labeled (a), (b), (c), and (d).]



33. $a_n = \frac{8}{n+1}$
34. $a_n = \frac{8n}{n+1}$
35. $a_n = 4(0.5)^{n-1}$
36. $a_n = n\left(2 - \frac{n}{10}\right)$

Finding the nth Term of a Sequence

In Exercises 37–50, write an expression for the apparent nth term (a_n) of the sequence. (Assume that n begins with 1.)

37. 3, 7, 11, 15, 19, . . .
38. 0, 3, 8, 15, 24, . . .
39. 3, 10, 29, 66, 127, . . .
40. 91, 82, 73, 64, 55, . . .
41. 1, −1, 1, −1, 1, . . .
42. 1, 3, 1, 3, 1, . . .
43. $-\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \frac{5}{6}, -\frac{6}{7}, \ldots$
44. $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \ldots$
45. $\frac{2}{1}, \frac{3}{3}, \frac{4}{5}, \frac{5}{7}, \frac{6}{9}, \ldots$
46. $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, \ldots$
47. $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots$
48. 2, 3, 7, 25, 121, . . .
49. $\frac{1}{1}, \frac{3}{1}, \frac{9}{2}, \frac{27}{6}, \frac{81}{24}, \ldots$
50. $\frac{2}{1}, \frac{6}{3}, \frac{24}{7}, \frac{120}{15}, \frac{720}{31}, \ldots$

Writing the Terms of a Recursive Sequence In Exercises 51–56, write the first five terms of the sequence defined recursively.

51. $a_1 = 28, \quad a_{k+1} = a_k - 4$
52. $a_1 = 3, \quad a_{k+1} = 2(a_k - 1)$
53. $a_1 = 81, \quad a_{k+1} = \frac{1}{3}a_k$
54. $a_1 = 14, \quad a_{k+1} = (-2)a_k$
55. $a_0 = 1, \quad a_1 = 2, \quad a_k = a_{k-2} + \frac{1}{2}a_{k-1}$
56. $a_0 = -1, \quad a_1 = 1, \quad a_k = a_{k-2} + a_{k-1}$

Fibonacci Sequence In Exercises 57 and 58, use the Fibonacci sequence. (See Example 5.)

57. Write the first 12 terms of the Fibonacci sequence whose nth term is a_n and the first 10 terms of the sequence given by

$$b_n = \frac{a_{n+1}}{a_n}, \quad n \geq 1.$$

58. Using the definition for b_n in Exercise 57, show that b_n can be defined recursively by

$$b_n = 1 + \frac{1}{b_{n-1}}.$$

Writing the Terms of a Sequence Involving Factorials In Exercises 59–62, write the first five terms of the sequence. (Assume that n begins with 0.)

59. $a_n = \dfrac{5}{n!}$
60. $a_n = \dfrac{1}{(n+1)!}$
61. $a_n = \dfrac{(-1)^n(n+3)!}{n!}$
62. $a_n = \dfrac{(-1)^{2n+1}}{(2n+1)!}$

Simplifying a Factorial Expression In Exercises 63–66, simplify the factorial expression.

63. $\dfrac{4!}{6!}$
64. $\dfrac{12!}{4! \cdot 8!}$
65. $\dfrac{(n+1)!}{n!}$
66. $\dfrac{(2n-1)!}{(2n+1)!}$

Finding a Sum In Exercises 67–74, find the sum.

67. $\sum_{i=0}^{4} 3i^2$
68. $\sum_{k=1}^{4} 10$
69. $\sum_{j=3}^{5} \dfrac{1}{j^2 - 3}$
70. $\sum_{i=1}^{5} (2i - 1)$
71. $\sum_{k=2}^{5} (k+1)^2(k-3)$
72. $\sum_{i=1}^{4} [(i-1)^2 + (i+1)^3]$
73. $\sum_{i=1}^{4} \dfrac{i!}{2^i}$
74. $\sum_{j=0}^{5} \dfrac{(-1)^j}{j!}$

Finding a Sum In Exercises 75–78, use a graphing utility to find the sum.

75. $\sum_{k=0}^{4} \dfrac{(-1)^k}{k!}$
76. $\sum_{k=0}^{4} \dfrac{(-1)^k}{k+1}$
77. $\sum_{n=0}^{25} \dfrac{1}{4^n}$
78. $\sum_{n=0}^{10} \dfrac{n!}{2^n}$

Using Sigma Notation to Write a Sum In Exercises 79–88, use sigma notation to write the sum.

79. $\dfrac{1}{3(1)} + \dfrac{1}{3(2)} + \dfrac{1}{3(3)} + \cdots + \dfrac{1}{3(9)}$
80. $\dfrac{5}{1+1} + \dfrac{5}{1+2} + \dfrac{5}{1+3} + \cdots + \dfrac{5}{1+15}$
81. $\left[2\left(\frac{1}{8}\right) + 3\right] + \left[2\left(\frac{2}{8}\right) + 3\right] + \cdots + \left[2\left(\frac{8}{8}\right) + 3\right]$
82. $\left[1 - \left(\frac{1}{6}\right)^2\right] + \left[1 - \left(\frac{2}{6}\right)^2\right] + \cdots + \left[1 - \left(\frac{6}{6}\right)^2\right]$
83. $3 - 9 + 27 - 81 + 243 - 729$
84. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots - \frac{1}{128}$
85. $\dfrac{1^2}{2} + \dfrac{2^2}{6} + \dfrac{3^2}{24} + \dfrac{4^2}{120} + \cdots + \dfrac{7^2}{40{,}320}$
86. $\dfrac{1}{1 \cdot 3} + \dfrac{1}{2 \cdot 4} + \dfrac{1}{3 \cdot 5} + \cdots + \dfrac{1}{10 \cdot 12}$
87. $\frac{1}{4} + \frac{3}{8} + \frac{7}{16} + \frac{15}{32} + \frac{31}{64}$
88. $\frac{1}{2} + \frac{2}{4} + \frac{6}{8} + \frac{24}{16} + \frac{120}{32} + \frac{720}{64}$

Finding a Partial Sum of a Series In Exercises 89–92, find the (a) third, (b) fourth, and (c) fifth partial sums of the series.

89. $\sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^i$
90. $\sum_{i=1}^{\infty} 2\left(\frac{1}{3}\right)^i$
91. $\sum_{n=1}^{\infty} 4\left(-\frac{1}{2}\right)^n$
92. $\sum_{n=1}^{\infty} 5\left(-\frac{1}{4}\right)^n$

Finding the Sum of an Infinite Series In Exercises 93–96, find the sum of the infinite series.

93. $\sum_{i=1}^{\infty} \dfrac{6}{10^i}$
94. $\sum_{k=1}^{\infty} \left(\dfrac{1}{10}\right)^k$
95. $\sum_{k=1}^{\infty} 7\left(\dfrac{1}{10}\right)^k$
96. $\sum_{i=1}^{\infty} \dfrac{2}{10^i}$