Note to students and teachers,

This is Nancy Stephenson's packet on differential equations in its entirety. This packet has been available in her Dropbox publicly for many years. Realize it does go beyond the scope of today's lesson from March 27, 2020 (7.3 Sketching Slope Fields – www.youtube.com/advancedplacement.) The lesson covered the material on pages 1-3 and much of page 4 should also be accessible. It is likely that we will refer to this document in subsequent lessons.

-Mark Kiraly & Virge Cornelius

Slope Fields
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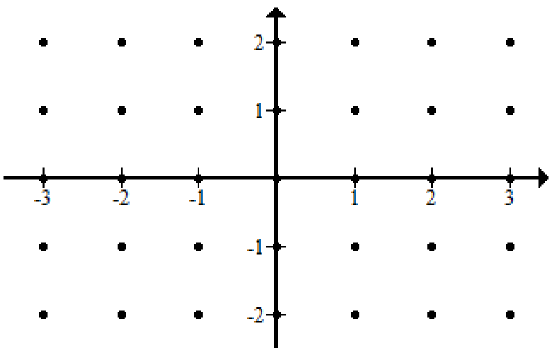
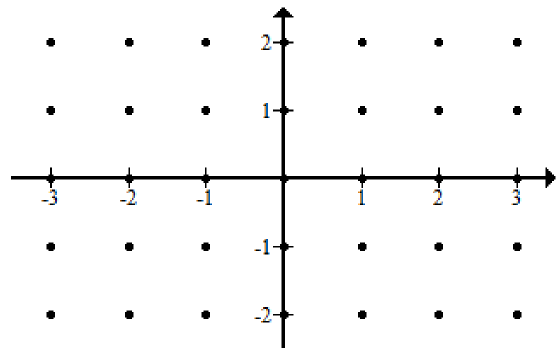
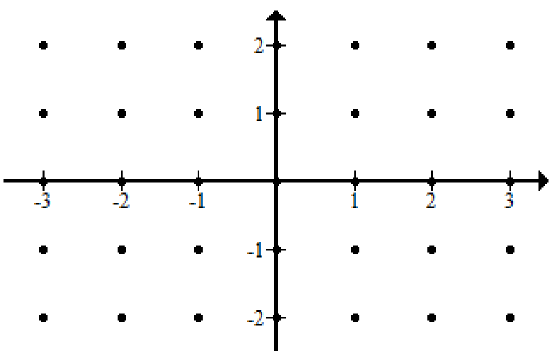
1. $\frac{dy}{dx} = x + 1$



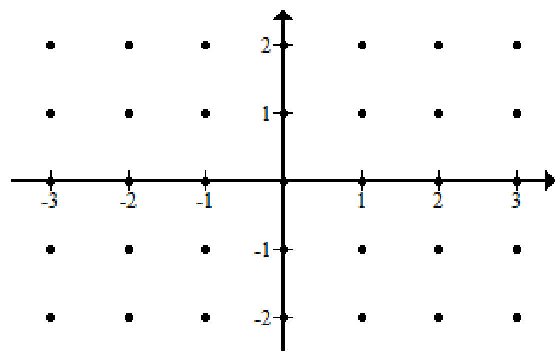
2. $\frac{dy}{dx} = 2y$



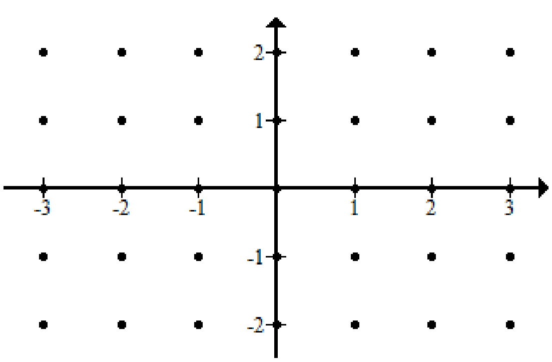
3. $\frac{dy}{dx} = x + y$



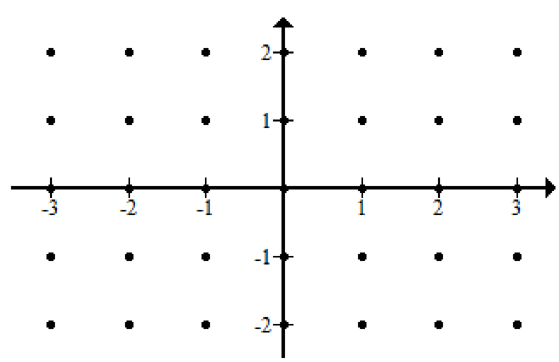
4. $\frac{dy}{dx} = 2x$



5. $\frac{dy}{dx} = y - 1$

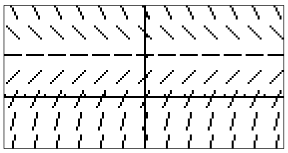


6. $\frac{dy}{dx} = -\frac{y}{x}$

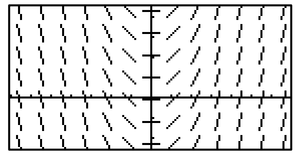


Match the slope fields with their differential equations.

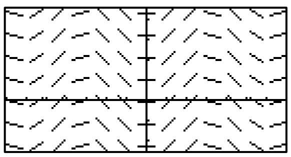
(A)



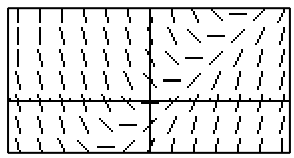
(B)



(C)



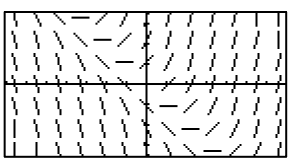
(D)



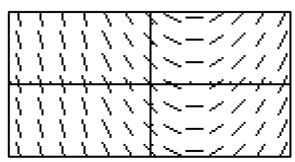
7. $\frac{dy}{dx} = \sin x$ 8. $\frac{dy}{dx} = x - y$ 9. $\frac{dy}{dx} = 2 - y$ 10. $\frac{dy}{dx} = x$

Match the slope fields with their differential equations.

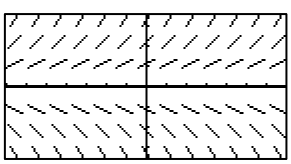
(A)



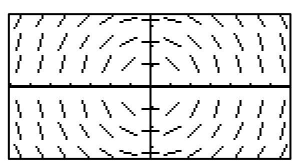
(B)



(C)

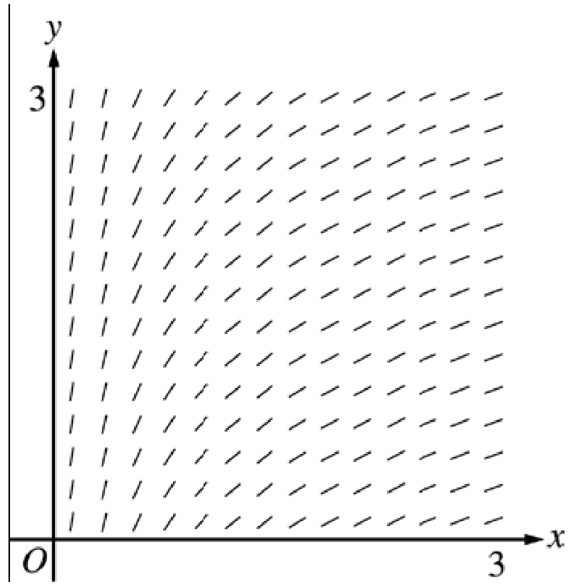


(D)



11. $\frac{dy}{dx} = 0.5x - 1$ 12. $\frac{dy}{dx} = 0.5y$ 13. $\frac{dy}{dx} = -\frac{x}{y}$ 14. $\frac{dy}{dx} = x + y$

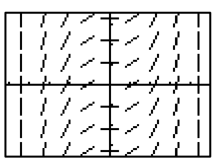
From the May 2008 *AP Calculus Course Description:*
15.



The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$ (B) $y = e^x$ (C) $y = e^{-x}$ (D) $y = \cos x$ (E) $y = \ln x$

16.

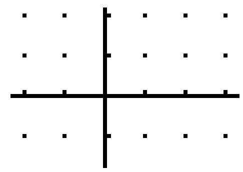


The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = \sin x$ (B) $y = \cos x$ (C) $y = x^2$ (D) $y = \frac{1}{6}x^3$ (E) $y = \ln x$

17. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(A) On the axes provided, sketch a slope field for the given differential equation.



(B) Let *f* be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point (1, 1). Then use your tangent line equation to estimate the value of $f(1.2)$.

(C) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$. Use your solution to find $f(1.2)$.

(D) Compare your estimate of $f(1.2)$ found in part (b) to the actual value of $f(1.2)$ found in part

(E) Was your estimate from part (b) an underestimate or an overestimate? Use your slope field to explain why.

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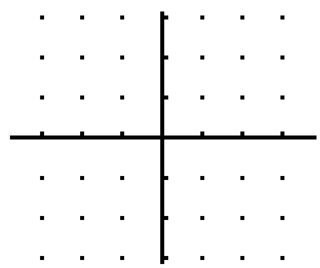
18. Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

(A) On the axes provided, sketch a slope field for the given differential equation.



(B) Sketch a solution curve that passes through the point (0, 1) on your slope field.

(C) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = 1$.

(D) Sketch a solution curve that passes through the point $(0, -1)$ on your slope field.

(E) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = -1$.

Slope Fields Worksheet Solutions

7. C
8. D
9. A
10. B
11. B
12. C
13. D
14. A
15. E
16. D
17. (B) Tangent line: $y-1=\frac{1}{2}(x-1)$

$f(1.2)\approx 1.1$

(C) $y=e^{\frac{x^2-1}{4}}$

$f(1.2)=1.116$

(D) The estimate from part (b) was an underestimate. Since the graph of $y=e^{\frac{x^2-1}{4}}$ is concave up, the tangent line found in part (b) lies below the curve.

18. (C) $y=\sqrt{x^2+1}$

(E) $y=-\sqrt{x^2+1}$