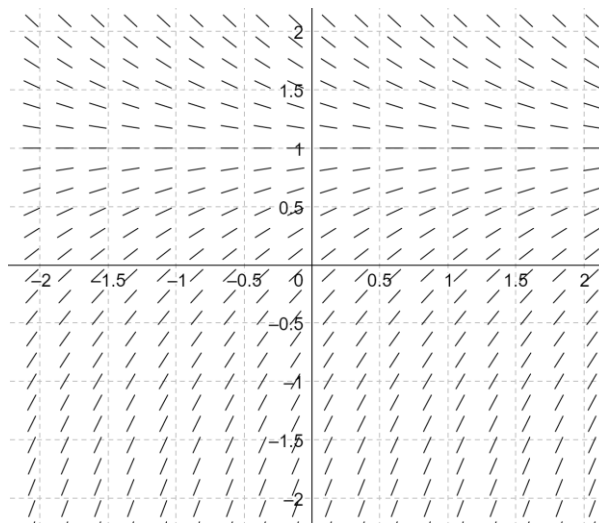


Big 10: Differential Equations and Slope Fields

Instructions: In the box below are the numbers 0 – 9. Complete the following and cross off the number for each answer. If you complete all problems correctly, you will cross off each number exactly once!

0	1	2	3	4	5	6	7	8	9
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- A) Consider the differential equation $\frac{dy}{dx} = \frac{3y^2}{\cos(\pi x)}$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(3) = -1$. The equation of the line tangent to the graph of f at $(3, -1)$ can be written as $y = mx + b$. Find b .



- B) A slope field for a given differential equation is shown above. There is a horizontal line with equation $y = c$ that satisfies the differential equation. Find the value of c .

C) Consider the differential equation $\frac{dy}{dx} = \frac{2y}{x^2}$. Let $y = h(x)$ be the particular solution to the differential equation through $\left(2, \frac{5}{e}\right)$. Find $\lim_{x \rightarrow \infty} h(x)$.

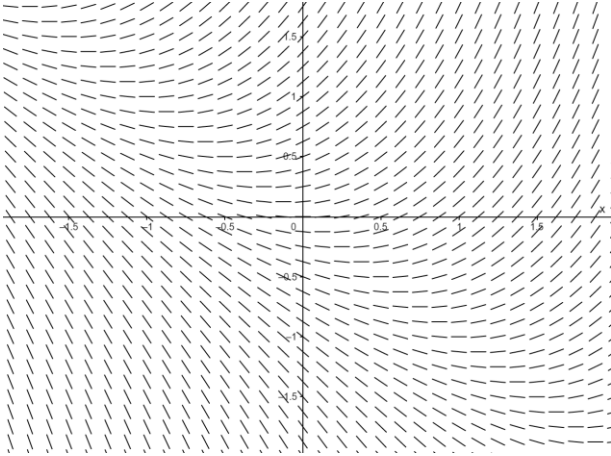
D) The differential equation $\frac{dy}{dx} = \frac{(3-x)^2}{y}$ has the particular solution $y = f(x)$ with initial condition $f(-1) = 4$. Find the slope of the tangent line to the graph of f at $x = -1$.

E) Let $y = g(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = \frac{3 \sin(\pi x)}{2y}$ with initial condition $g\left(\frac{1}{2}\right) = 3$. Then $g(x) = \sqrt{a + b \cos(\pi x)}$ where a and b are constants. Find a .

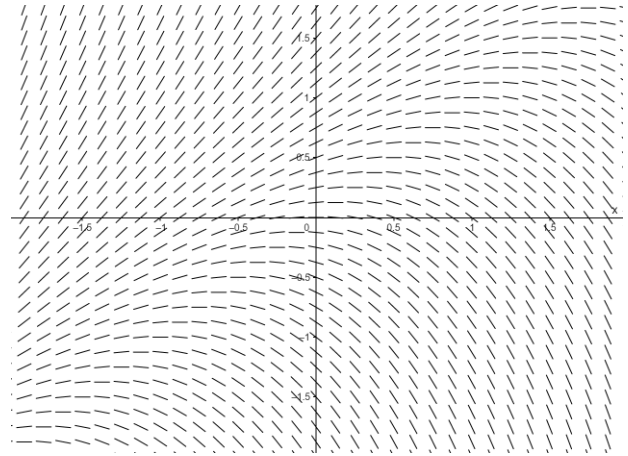
F) The non separable differential equation $\frac{dy}{dx} = 8x - 2y$ has a **linear** particular solution of the form $y = mx + b$. Find $m + b$.

G) Which of the following could be a slope field for the differential equation $\frac{dy}{dx} = x - y$?

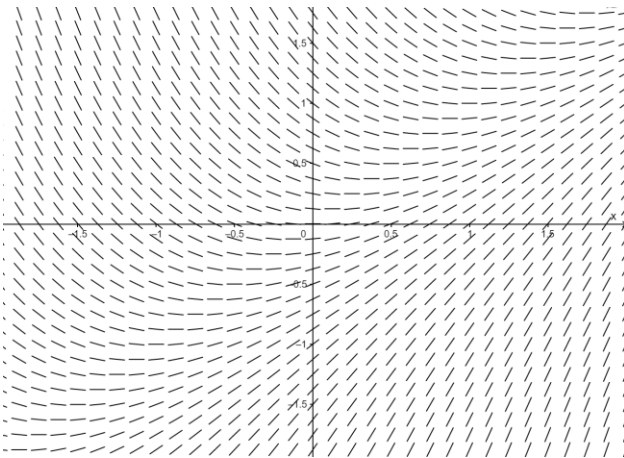
3.



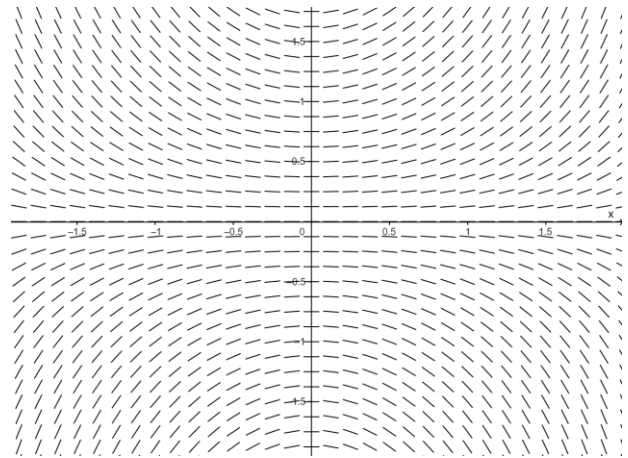
5.



7.



9.



H) Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{3}(9 - y)$ with the initial condition $f(1) = 3$. Use the line tangent to the graph of f at $(1, 3)$ to approximate $f(2.5)$.

I) For $0 \leq t \leq 3$ days, the number of weeds in a large garden is given by the function W that satisfies the differential equation $\frac{dW}{dt} = \frac{1}{12}(-318 + 24W)$. At time $t = 2$ days, there are 20 weeds in the garden. Find $\frac{d^2W}{dt^2}$ when $W = 14$.

J) Consider the differential equation $\frac{dy}{dx} = \left(1 - \frac{4}{x^3}\right)(y - 1)^2$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(2) = -1$. Find $f(1)$.

ANSWER KEY

A) 8

B) 1

C) 5

D) 4

E) 9

F) 2

G) 7

H) 6

I) 3

J) 0