5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

   (a) For $0 \le t \le 8$, when is particle P moving to the left?

   (b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

   (c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

   (d) Find the position of particle Q the first time it changes direction.

© 2017 The College Board.
Visit the College Board on the Web: www.collegeboard.org.

2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

   The particle is at position $x = -5$ at time $t = 0$.

   (a) Find the acceleration of the particle at time $t = 3$.

   (b) Find the position of the particle at time $t = 3$.

   (c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} \left|v(t)\right|\,dt$. Interpret the meaning of each integral in the context of the problem.

   (d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

© 2018 The College Board.
Visit the College Board on the Web: www.collegeboard.org.

AP® CALCULUS AB
2017 SCORING GUIDELINES

Question 5

(a) $x_P'(t) = \dfrac{2t-2}{t^2-2t+10} = \dfrac{2(t-1)}{t^2-2t+10}$

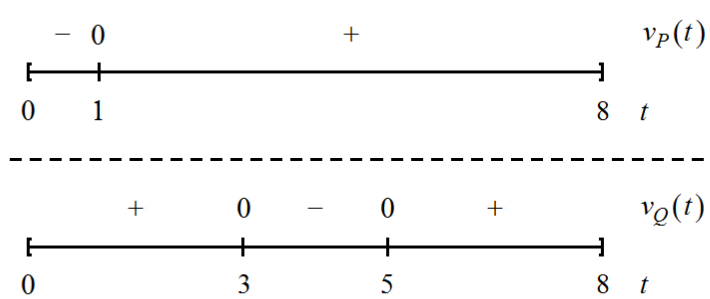
$t^2 - 2t + 10 > 0$ for all t.

$x_P'(t) = 0 \Rightarrow t = 1$
$x_P'(t) < 0$ for $0 \le t < 1$.

Therefore, the particle is moving to the left for $0 \le t < 1$.

$2 : \begin{cases} 1 : x_P'(t) \\ 1 : \text{interval} \end{cases}$

(b) $v_Q(t) = (t-5)(t-3)$
$v_Q(t) = 0 \Rightarrow t = 3, t = 5$



Both particles move in the same direction for $1 < t < 3$ and $5 < t \le 8$ since $v_P(t) = x_P'(t)$ and $v_Q(t)$ have the same sign on these intervals.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{analysis using } v_P(t) \text{ and } v_Q(t) \end{cases}$

Note: $1/2$ if only one interval with analysis

Note: $0/2$ if no analysis

(c) $a_Q(t) = v_Q'(t) = 2t - 8$
$a_Q(2) = 2 \cdot 2 - 8 = -4$

$a_Q(2) < 0$ and $v_Q(2) = 3 > 0$

At time $t = 2$, the speed of the particle is decreasing because velocity and acceleration have opposite signs.

$2 : \begin{cases} 1 : a_Q(2) \\ 1 : \text{speed decreasing with reason} \end{cases}$

(d) Particle Q first changes direction at time $t = 3$.

$$x_Q(3) = x_Q(0) + \int_0^3 v_Q(t)\,dt = 5 + \int_0^3 \left(t^2 - 8t + 15\right) dt$$

$$= 5 + \left[\frac{1}{3}t^3 - 4t^2 + 15t\right]_{t=0}^{t=3} = 5 + (9 - 36 + 45) = 23$$

$3 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

© 2017 The College Board.
Visit the College Board on the Web: www.collegeboard.org.

Question 2

(a) $v'(3) = -2.118$

The acceleration of the particle at time $t = 3$ is -2.118.

1 : answer

(b) $x(3) = x(0) + \int_0^3 v(t)\,dt = -5 + \int_0^3 v(t)\,dt = -1.760213$

The position of the particle at time $t = 3$ is -1.760.

$$3 : \begin{cases} 1 : \int_0^3 v(t)\,dt \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$$

(c) $\int_0^{3.5} v(t)\,dt = 2.844$ (or 2.843)

$\int_0^{3.5} |v(t)|\,dt = 3.737$

The integral $\int_0^{3.5} v(t)\,dt$ is the displacement of the particle over the time interval $0 \le t \le 3.5$.

The integral $\int_0^{3.5} |v(t)|\,dt$ is the total distance traveled by the particle over the time interval $0 \le t \le 3.5$.

$$3 : \begin{cases} 1 : \text{answers} \\ 2 : \text{interpretations of } \int_0^{3.5} v(t)\,dt \\ \quad \text{and } \int_0^{3.5} |v(t)|\,dt \end{cases}$$

(d) $v(t) = x_2'(t)$

$v(t) = 2t - 1 \Rightarrow t = 1.57054$

The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).

$$2 : \begin{cases} 1 : \text{sets } v(t) = x_2'(t) \\ 1 : \text{answer} \end{cases}$$

© 2018 The College Board.
Visit the College Board on the Web: www.collegeboard.org.