Question 6

(a) $\int_0^2 L(t)\,dt = \int_0^2 (-2t + 15)\,dt = \left[-t^2 + 15t\right]_0^2$

$= -4 + 30 = 26$

26 hundred bees leave the hive during the time interval $0 \le t \le 2$.

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) The total number of bees, in hundreds, in the hive at time t is

$35 + \int_0^t (E(x) - L(x))\,dx.$

$$\begin{aligned} 35 + \int_0^4 (E(x) - L(x))\,dx &= 35 + \int_0^4 \left(\left(16x - 3x^2\right) - (-2x + 15)\right) dx \\ &= 35 + \int_0^4 \left(-3x^2 + 18x - 15\right) dx \\ &= 35 + \left[-x^3 + 9x^2 - 15x\right]_0^4 \\ &= 35 + (-64 + 144 - 60) \\ &= 55 \end{aligned}$$

55 hundred bees are in the hive at time $t = 4$.

$3 : \begin{cases} 1 : \text{expression for total} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) Let $B(t)$ be the total number of bees, in hundreds, in the hive at time t, for $0 \le t \le 4$.

$B(t) = 35 + \int_0^t (E(x) - L(x))\,dx$

$B'(t) = E(t) - L(t) = -3t^2 + 18t - 15 = -3(t - 1)(t - 5)$

$B'(t) = 0 \Rightarrow t = 1, t = 5$

t	$B(t)$
0	35
1	28
4	55

The minimum numbers of bees in the hive for $0 \le t \le 4$ is 28 hundred bees.

$3 : \begin{cases} 1 : \text{sets } E(t) - L(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

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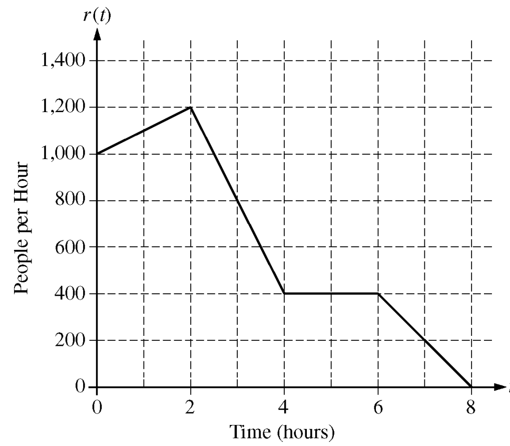
Question 3

There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.



(a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

(b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

(c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

(d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

(a) $\int_0^3 r(t)\,dt = 2\cdot\frac{1000+1200}{2} + \frac{1200+800}{2} = 3200$ people	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) The number of people waiting in line is increasing because people move onto the ride at a rate of 800 people per hour and for $2 < t < 3$, $r(t) > 800$.	1 : answer with reason
(c) $r(t) = 800$ only at $t = 3$ For $0 \le t < 3$, $r(t) > 800$. For $3 < t \le 8$, $r(t) < 800$. Therefore, the line is longest at time $t = 3$. There are $700 + 3200 - 800\cdot 3 = 1500$ people waiting in line at time $t = 3$.	$3 : \begin{cases} 1 : \text{identifies } t = 3 \\ 1 : \text{number of people in line} \\ 1 : \text{justification} \end{cases}$
(d) $0 = 700 + \int_0^t r(s)\,ds - 800t$	$3 : \begin{cases} 1 : 800t \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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Question 4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \dfrac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

(a) Average acceleration of rocket A is

$$\frac{v(80) - v(0)}{80 - 0} = \frac{49 - 5}{80} = \frac{11}{20} \text{ ft/sec}^2$$

1 : answer

(b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds.

A midpoint Riemann sum is

$$20[v(20) + v(40) + v(60)]$$
$$= 20[22 + 35 + 44] = 2020 \text{ ft}$$

3 : { 1 : explanation; 1 : uses $v(20)$, $v(40)$, $v(60)$; 1 : value }

(c) Let $v_B(t)$ be the velocity of rocket B at time t.

$$v_B(t) = \int \frac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1} + C$$
$$2 = v_B(0) = 6 + C$$
$$v_B(t) = 6\sqrt{t+1} - 4$$
$$v_B(80) = 50 > 49 = v(80)$$

Rocket B is traveling faster at time $t = 80$ seconds.

4 : { 1 : $6\sqrt{t+1}$; 1 : constant of integration; 1 : uses initial condition; 1 : finds $v_B(80)$, compares to $v(80)$, and draws a conclusion }

Units of ft/sec^2 in (a) and ft in (b)

1 : units in (a) and (b)

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Question 2

(a) $\int_0^2 f(t)\,dt = 20.051175$

20.051 pounds of bananas are removed from the display table during the first 2 hours the store is open.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $f'(7) = -8.120$ (or -8.119)

After the store has been open 7 hours, the rate at which bananas are being removed from the display table is decreasing by 8.120 (or 8.119) pounds per hour per hour.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{meaning} \end{cases}$

(c) $g(5) - f(5) = -2.263103 < 0$

Because $g(5) - f(5) < 0$, the number of pounds of bananas on the display table is decreasing at time $t = 5$.

$2 : \begin{cases} 1 : \text{considers } f(5) \text{ and } g(5) \\ 1 : \text{answer with reason} \end{cases}$

(d) $50 + \int_3^8 g(t)\,dt - \int_0^8 f(t)\,dt = 23.347396$

23.347 pounds of bananas are on the display table at time $t = 8$.

$3 : \begin{cases} 2 : \text{integrals} \\ 1 : \text{answer} \end{cases}$

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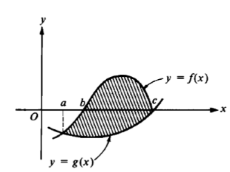
1. A region in the plane is bounded by the graph of y = 1/x , the *x*-axis, the line *x* = *m*, and the line *x* = 2*m*, *m* > 0. The area of this region

- is independent of *m*. ✔
- increases as *m* increases.
- decreases as *m* increases.
- decreases as *m* increases when m < 1/2 ; increases as *m* increases when m > 1/2.
- increases as *m* increases when m < 1/2 ; decreases as *m* increases when m >1/2.

2.



The area of the shaded region in the figure above is represented by which of the following integrals?

- $\int_a^c (|f(x)| - |g(x)|)dx$
- $\int_b^c f(x)dx - \int_a^c g(x)dx$
- $\int_a^c (g(x) - f(x))dx$
- $\int_a^c (f(x) - g(x))dx$
- $\int_a^b (g(x) - f(x))dx + \int_b^c (f(x) - g(x))\,dx$