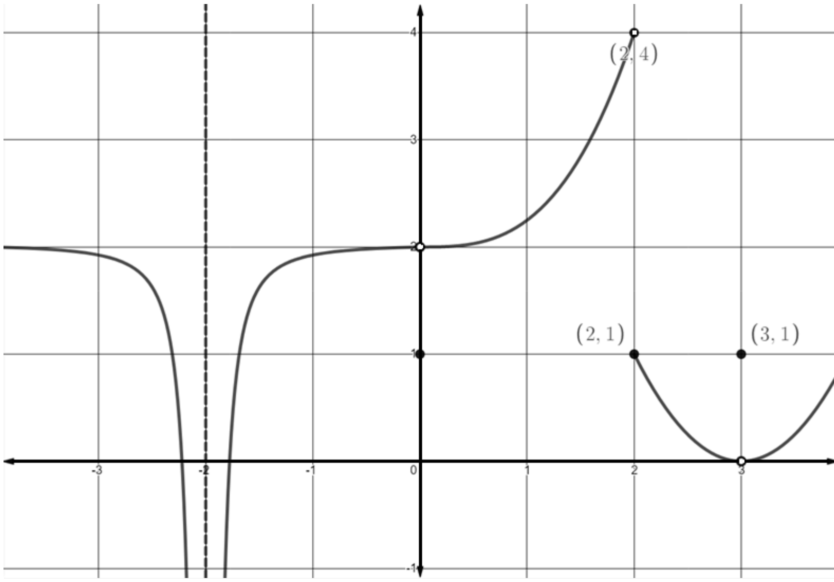
From AP Live Calculus AB - April 13; Unit 1: Limits and Continuity

1-3: Use the graph of the piecewise function, f, shown to find the requested values.



1. $f(0) = 1$

2. $\lim_{x \to 2^-} f(x) = 4$

3. $\lim_{x \to 3} f(x) = 0$

4. $\lim_{x \to -5} \frac{x+5}{x^2-25} = -\frac{1}{10}$

5. $\lim_{x \to 3} \frac{\sqrt{2x+3}-3}{x-3} = \frac{1}{3}$

6. Find the horizontal asymptote(s) of $f(x) = \frac{\sqrt{4x^2+7}}{3x-5}$.

2 horizontal asymptotes. $y = \frac{2}{3}$ & $y = -\frac{2}{3}$

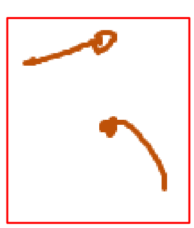
A function f is said to be continuous at $x = a$ if and only if…

- $f(a)$ exists
- $\lim_{x \to a} f(x) = L$ (exists); may also show $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$
- $\lim_{x \to a} f(x) = L = f(a)$

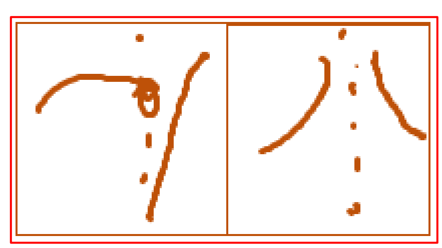
Draw an example(s) of each type of discontinuity.

Removable

Jump



Infinite



7. Find a value or values of q that will make $g(x)$ continuous. $g(x) = \begin{cases} qx + 3 & \text{for } x \leq 1 \\ (x+q)^2 - 10 & \text{for } x > 1 \end{cases}$

$q = 3$ or $q = -4$

8. Given selected values of the continuous function $h(x)$, what is the fewest number of times $h(x)$ is 43 on the interval $[0, 30]$?

x	0	5	10	15	20	25	30
$h(x)$	100	40	40	110	30	10	50

4 times

9. Given that $\lim_{x\to 7} v(x) = 6$, $\lim_{x\to -7} v(x) = 3$, and $\lim_{x\to 7} c(x) = 5$ and that $v(x)$and $c(x)$ are continuous,

evaluate the following. $\lim_{x\to 7^+} [\, v(-x) - 5c(x)]$

$\lim_{x\to 7^+} [\, v(-x) - 5c(x)] = \lim_{x\to 7} [\, v(-x) - 5c(x)] = -22.$ Note: If f is continuous, then $\lim_{x\to a} f(-x) = \lim_{x\to -a} f(x)$.

10. Find the limit of $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$.

$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$ (via squeeze theorem)

11. Tram α runs back and forth on a straight north-south path between an amusement park and an adjacent parking lot. Tram α's velocity, measured in meters per second, is given by the continuous function $v_\alpha(t)$ where time t is measured in seconds. Selected values for $v_\alpha(t)$ are given in the table above.

t (seconds)	0	40	100	160	240
$v_\alpha(t)$ (meters/second)	0	10	4	-12	-15

Do the data in the table support the conclusion that tram α's velocity is -10 meters per second at some time with $40 < t < 100$? Give a reason for your answer. No. By IVT we can only guarantee values between 10 and 4.

12. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is continuous and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table below.

t (hours)	0	1	3	6	8
$R(t)$ (liters/hour)	1340	1190	950	740	700

For $0 \leq t \leq 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

Looking for $W(a) = R(a)$ or $W(a) - R(a) = 0$.

$W(t) - R(t)$ is continuous.

$W(0) - R(0) = 2000 - 1340 > 0$

$W(8) - R(8) = 81.5244 - 700 < 0$

Therefore there exists an a in (0, 8) such that $W(a) - R(a) = 0$ and $W(a) = R(a)$.

13. The velocity of particle, P, moving along the x-axis is given by the differentiable function v_p, where $v_p(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_p(t)$ are shown in the table below. Particle P is at the origin at time $t = 0$.

t (hours)	0	0.3	1.7	2.8	4
$v_p(t)$ (meters per hour)	0	55	-29	55	48

Justify why there must be at least one time t, for $0.3 \leq t \leq 2.8$, at which $v_p{}'(t)$, the acceleration of particle P, equals 0 meters per hour per hour.

Disclosure statement: can't be done without mean value theorem. Trickery and foreshadowing for unit 5!

14. Let g and h be continuous functions such that $g(5) = h(5) = 1.$ Let k be a function satisfying

$g(x) \leq k(x) \leq h(x)$ for $3 < x < 7$. Is k continuous at $x = 5$? Justify your answer.

By given: $g(5) \leq k(5) \leq h(5)$, so $k(5) = 1$.

Since g and h are continuous on (3, 7) and $5 \in (3,7)$, then $\lim_{x\to 5} g(x) = g(5) = 1$ and $\lim_{x\to 5} h(x) = h(5) = 1$

On (3, 7): $\lim_{x\to 5} g(x) \leq \lim_{x\to 5} k(x) \leq \lim_{x\to 5} h(x)$

$1 \leq \lim_{x\to 5} k(x) \leq 1$ and $\lim_{x\to 5} k(x) = 1 = k(5)$ and k is continuous and $k = 5$.