Derivative Rules: Level 2

Analytical

$$\frac{1}{2}x^2 - \frac{1}{2}xy^2 + 6y = 2x$$

Find $\frac{dy}{dx}$ when $x = 4$

$$x - \frac{1}{2}y^2 - xyy' + 6y' = 2$$

$$-xyy' + 6y' = 2 - x + \frac{1}{2}y^2$$

$$y'(-xy + 6) = 2 - x + \frac{1}{2}y^2$$

$$\frac{dy}{dx} = \frac{2 - x + \frac{1}{2}y^2}{-xy + 6}$$

When $x = 4$: $\frac{1}{2}(4)^2 - \frac{1}{2}(4)y^2 + 6y = 2(4)$

$8 - 2y^2 + 6y = 8$

$-2y^2 + 6y = 0$ So (4, 0) and (4, 3)

$$\left.\frac{dy}{dx}\right|_{(4,0)} = \frac{2 - 4 + \frac{1}{2}(0)^2}{-4(0) + 6} = -\frac{1}{3}$$

$$\left.\frac{dy}{dx}\right|_{(4,3)} = \frac{2 - 4 + \frac{1}{2}(3)^2}{-4(3) + 6} = -\frac{\frac{5}{2}}{36} = -\frac{5}{12}$$

Numerical

x	-1	0	2	5
$f(x)$	0	2	5	4
$f'(x)$	-3	4	1	-2

$$g(x) = f\big(f(x)\big)$$

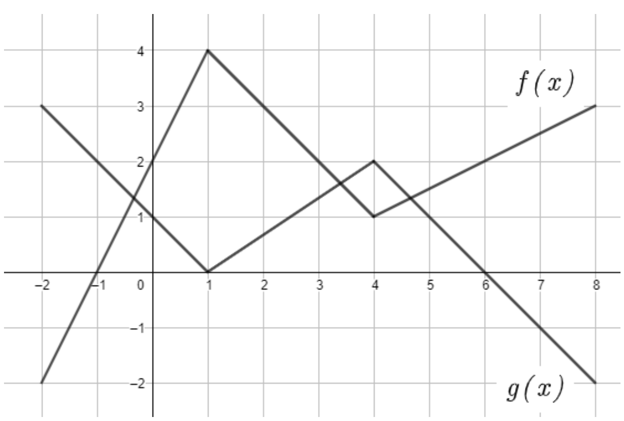
$$g'(-1) =$$

$$g'(x) = f'\big(f(x)\big)f'(x)$$

$$g'(-1) = f'(f(-1)f'(-1)$$

$$f'(0)f'(-1) = (4)(-3) = -12$$

Graphical



$$h(x) = g(f(x))$$
$$h'(2) =$$

$$h'(x) = g'\big(f(x)\big)f'(x)$$

$$h'(2) = g'\big(f(2)\big)f'(2) = g'(3)f'(2)$$

$$= \frac{2}{3} \cdot (-1) = -\frac{2}{3}$$

Conceptual/Verbal

g is a linear function with $g(3) = g'(3) = 4$

$$k(x) = \frac{g(x)}{g(x+2)}$$

$$k'(3) =$$

$$k'(x) = \frac{g'(x)g(x+2) - g'(x+2)(1)g(x)}{[g(x+2)]^2}$$

$$g(x) = 4 + 4(x-3) = 4x - 8; \quad g'(x) = 4$$

$$g(3) = g'(3) = 4$$

$$g(3+2) = g(5) = 4(5) - 8 = 12$$

$$k'(3) = \frac{g'(3)g(3+2) - g'(3+2)(1)g(3)}{[g(3+2)]^2}$$

$$\frac{4(12) - (4)(1)(4)}{[12]^2} = \frac{32}{144} = \frac{2}{9}$$