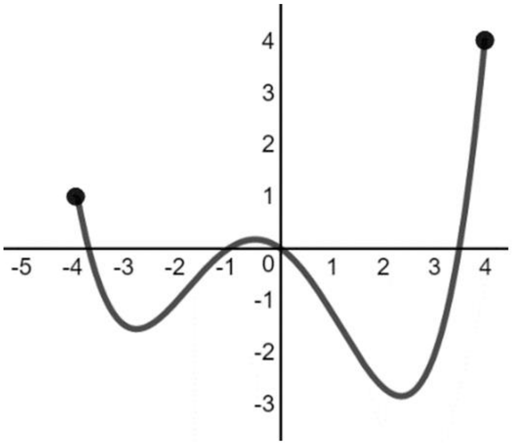
UNIT 5 REVIEW: Analytical Applications of Differentiation AP Calc AB April 20, 2020

NOTE: This is not a comprehensive review. Some topics, such as exploring behaviors of implicit relations, have already been touched on and others, such as extreme values, we will highlight in the Unit 6 review.

WARM UP:

The graph of a differentiable function g is shown below on the closed interval $[-4, 4]$. How many values of x in the open interval $(-4, 4)$ satisfy the conclusion of the Mean Value Theorem for g on $[-4, 4]$?



Mean Value Theorem:

Answer:

Now, given $g(x)$, the same function shown above, has x-intercepts on the interval $[-4, 4]$ at $x = -3.8$, $x = -1, x = 0,$ and $x = 3.4$. Also, $g(x)$ has horizontal tangents at $x = -2.9, x = -0.5,$ and $x = 2.2$. Finally, let $h(x)$ be a twice differentiable function such that $h'(x) = g(x)$. Whew.

Determine the following about the function $h(x)$ on the open interval $(-4, 4)$. Give your reasoning for each.

1. On what open interval(s) is $h(x)$ increasing?

2. On what open interval(s) is $h(x)$ decreasing?

3. On $(-4, 4)$, what are the x-coordinates of each local (relative) maximum on $h(x)$?

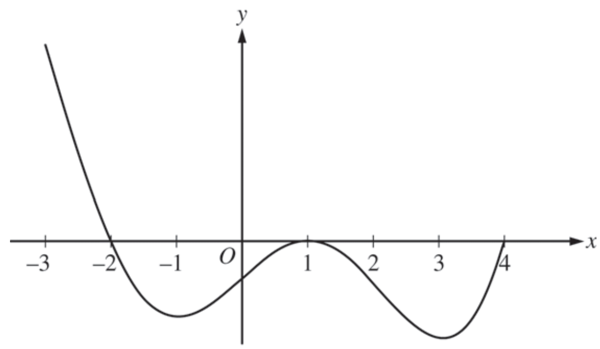
4. On $(-4, 4)$, what are the x-coordinates of each local (relative) minimum on $h(x)$?

5. On what open interval(s) is $h(x)$ concave up?

6. On what open interval(s) is $h(x)$ concave down?

7. What are the x-coordinates of each inflection point on $h(x)$?

2015 AB 5



Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

   (a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

   (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

   (c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

   (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

2019 AB

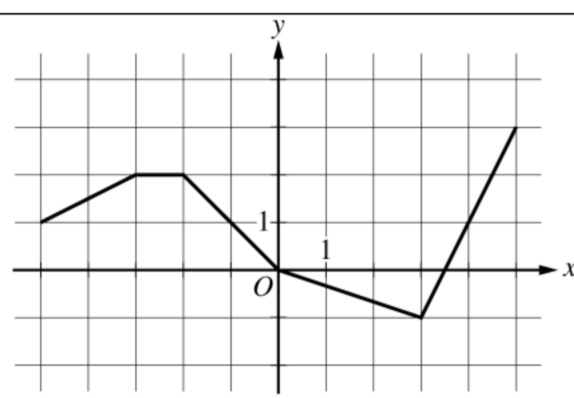
t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	−29	55	48

2. The velocity of a particle, P, moving along the x-axis is given by the differentiable function v_P, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.

   (a) Justify why there must be at least one time t, for $0.3 \le t \le 2.8$, at which $v_P'(t)$, the acceleration of particle P, equals 0 meters per hour per hour.

2017 AB

x	$g(x)$	$g'(x)$
−5	10	−3
−4	5	−1
−3	2	4
−2	3	1
−1	1	−2
0	0	−3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.
   Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.
   Let h be the function whose graph, consisting of five line segments, is shown in the figure above.
   (d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.