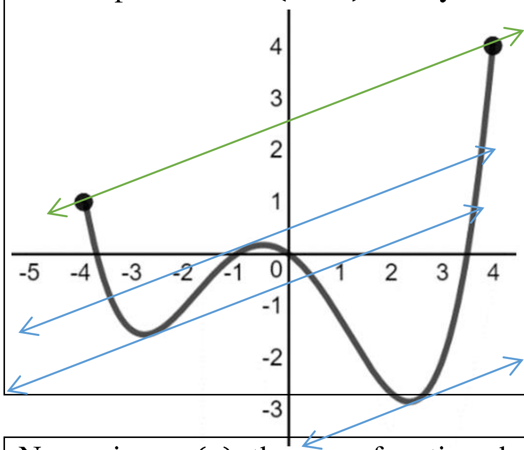
UNIT 5 REVIEW: Analytical Applications of Differentiation AP Calc AB April 20, 2020

NOTE: This is not a comprehensive review. Some topics, such as exploring behaviors of implicit relations, have already been touched on and others, such as extreme values, we will highlight in the Unit 6 review.

WARM UP:

The graph of a differentiable function g is shown below on the closed interval $[-4, 4]$. How many values of x in the open interval $(-4, 4)$ satisfy the conclusion of the Mean Value Theorem for g on $[-4, 4]$?



Mean Value Theorem: Given a function is continuous on $[a, b]$ and differentiable on (a, b). There must be a value $x = c$ in the interval (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Answer: 3 times

Now, given $g(x)$, the same function shown above, has x-intercepts on the interval $[-4, 4]$ at $x = -3.8$, $x = -1, x = 0,$ and $x = 3.4$. Also, $g(x)$ has horizontal tangents at $x = -2.9, x = -0.5,$ and $x = 2.2$. Finally, let $h(x)$ be a twice differentiable function such that $h'(x) = g(x)$. Whew.

Determine the following about the function $h(x)$ on the open interval $(-4, 4)$. Give your reasoning for each.

1. On what open interval(s) is $h(x)$ increasing? h is increasing on $(-4, -3.4), (-1, 0),$ and $(3.4, 4)$. Because $g = h'$ is positive on $(-4, -3.4), (-1, 0),$ and $(3.4, 4)$.

2. On what open interval(s) is $h(x)$ decreasing? h is decreasing on $(-3.4, -1)$ and $(0, 3.4)$. Because g=h' is negative on $(-3.4, -1)$ and $(0, 3.4)$.

3. On $(-4, 4)$, what are the x-coordinates of each local (relative) maximum on $h(x)$? h has a relative maximum at both $x = -3.8$ and $x = 0$ because $g = h'$ changes from positive to negative at those two values.

4. On $(-4, 4)$, what are the x-coordinates of each local (relative) minimum on $h(x)$? h has a relative minimum at both $x = -1$ and $x = 3.4$ because $g = h'$ changes from negative to positive at those two values.

5. On what open interval(s) is $h(x)$ concave up? h is concave up on $(-2.9, -0.5)$ and $(2.2, 4)$ because $g = h'$ is increasing on those intervals. Alternative: h is concave up on $(-2.9, -0.5)$ and $(2.2, 4)$ because h'' is positive on those intervals. $h'' > 0$ is shown by $g = h'$ increasing on those intervals.

6. On what open interval(s) is $h(x)$ concave down? h is concave down on $(-4, -2.9)$ and $(-0.5, 2.2)$ because $g = h'$ is decreasing on those intervals. Alternative: h is concave down on $(-4, -2.9)$ and $(-0.5, 2.2)$ because h'' is negative on those intervals. $h'' < 0$ is shown by $g = h'$ decreasing on those intervals.

7. What are the x-coordinates of each inflection point on $h(x)$? $x = -2.9, x = -0.5,$ and $x = 2.2$. Because those are the locations of relative minimums and maximums of $g = h'$. Alternative: Because $g' = h''$ changes from increasing to decreasing and vise versa at those locations.

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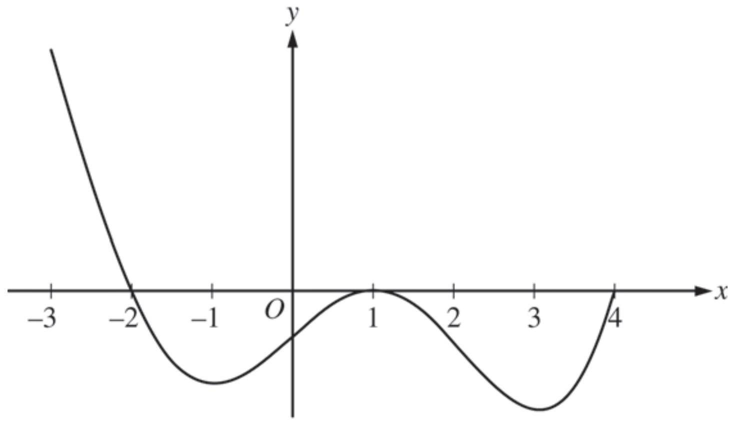

Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

   (a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

   (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

   (c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

   (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

(a) $f'(x) = 0$ at $x = -2$, $x = 1$, and $x = 4$. $f'(x)$ changes from positive to negative at $x = -2$. Therefore, f has a relative maximum at $x = -2$.	$2 : \begin{cases} 1 : \text{identifies } x = -2 \\ 1 : \text{answer with reason} \end{cases}$
(b) The graph of f is concave down and decreasing on the intervals $-2 < x < -1$ and $1 < x < 3$ because f' is decreasing and negative on these intervals.	$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$
(c) The graph of f has a point of inflection at $x = -1$ and $x = 3$ because f' changes from decreasing to increasing at these points. The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing at this point.	$2 : \begin{cases} 1 : \text{identifies } x = -1, 1, \text{and } 3 \\ 1 : \text{reason} \end{cases}$
(d) $f(x) = 3 + \int_1^x f'(t)\,dt$ $f(4) = 3 + \int_1^4 f'(t)\,dt = 3 + (-12) = -9$ $f(-2) = 3 + \int_1^{-2} f'(t)\,dt = 3 - \int_{-2}^1 f'(t)\,dt$ $= 3 - (-9) = 12$	$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{expression for } f(x) \\ 1 : f(4) \text{ and } f(-2) \end{cases}$