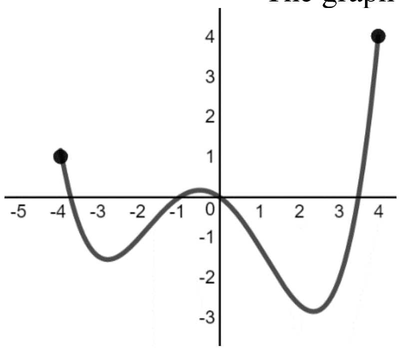
UNIT 6: Integration and Accumulation of Change   AP Calc AB April 21, 2020

Example 1: Extreme Values / Exploring Accumulations of Change



The graph of a differentiable function $\mathrm{h}(x)$ is shown below on the closed interval $[-4, 4]$. The function $h(x)$ has x-intercepts on the interval $[-4, 4]$ at $x = -3.8$, $x = -1, x = 0$, and $x = 3.4$. Also, $h(x)$ has horizontal tangents at $x = -2.9, x = -0.5$, and $x = 2.2$. The areas of the regions bounded by the x-axis and the graph of $h(x)$ on the intervals $[-1, 0]$, $[0, 3.4]$ and $[3.4, 4]$ are 0.3, 12, and 2, respectively.

For all x, $g(x) = 11 + \int_0^x h(t)dt$.

(a) Find: $g(3.4)$ $\quad g(4)$ $\quad g(-1)$ $\quad g'(3.4)$ $\quad g''(2.2)$

$11 - 12 = -1$ $\quad -1 + 2 = 1$ $\quad 11 - (0.3) = 10.7$ $\quad h(3.4) = 0$ $\quad h'(2.2) = 0$

(b) Find the maximum and minimum of $g(x)$ on the interval $[-1, 4]$. Explain your answers.

$g(-1) = 10.7; g(0) = 11; g(3.4) = -1, g(4) = 1$ are only candidates (endpoints and where $g' = 0$).

Minimum is $g(3.4) = -1$. Maximum is $g(0) = 11$.

Example 2: Approximating areas with Riemann Sums

Little is known about the function $v(t)$ except selected values given in the table. Use a ______________ sum with the four sub intervals indicated by the data in the table to approximate $\int_{-2}^{10} v(t)dt$.

t	-2	1	6	7	10
$v(t)$	3	4	-9	2	0

(a) left Riemann

$(1 - (-2))(3) + (6 - 1)(4) + (7 - 6)(-9) + (10 - 7)(2)$

$9 + 20 + (-9) + 6$

26

(b) right Riemann

$(1 - (-2))(4) + (6 - 1)(-9) + (7 - 6)(2) + (10 - 7)(0)$

$12 + (-45) + 2$

-31

(c) trapezoidal

{the average of the left and right Riemann Sums}

$(26 + (-31))/2 = -5/2$

Note: We did an example of a midpoint sum in one of the Unit 8 videos. ☺

Example 3: Fundamental Theorem of Calculus

(a) $\int_4^9 \frac{1}{2\sqrt{x}} dx =$

(b) $\frac{d}{dx}\left[\int_a^{x^3} \frac{1}{t^2+t-5} dt\right] =$

$\sqrt{x}\Big|_4^9 = 3 - 2 = 1$

$\frac{1}{(x^3)^2+(x^3)-5} \cdot 3x^2 = \frac{3x^2}{x^6+x^3-5}$

Example 4: Properties of Definite Integrals

Given $\int_3^6 f(x)dx = 7.5$, $\int_5^3 f(x)dx = 2$, and $\int_6^3 g(x)dx = -12$, determine:

(a) $\int_5^6 f(x)dx$

(b) $\int_3^6 g(x)dx$

(c) $\int_3^6 [4f(x) - 2g(x) + 4]dx$

$7.5 - (-2) = 9.5$

$-(-12) = 12$

$4(7.5) - 2(12) + 4(3) = 30 - 24 + 12 = 18$

Example 5: Integration Blast! (Separate paper probably required!)

(a) $\int sin\,\theta d\theta$

(b) $\int \frac{1-x}{1+x^2} dx$

$-\cos\theta + C$

$\int \frac{1}{1+x^2} dx - \frac{1}{2}\int \frac{2x}{1+x^2} dx$; $u = 1 + x^2; du = 2x\,dx$

$\arctan x - \frac{1}{2}\ln|1 + x^2| + C$

(c) $\int \frac{1}{(2-G)^{2/3}} dG$

(d) $\int_{-5}^{5} \sqrt{25 - x^2} dx$

$-3(2-G)^{\frac{1}{3}} + C$

$\frac{\pi(5)^2}{2}$ (semi circle of radius 5)

Remember: What's the integral of $\frac{1}{cabin}$ with respect to *cabin*? House Boat! $\ln|cabin| + sea$