12.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. If $f(x)$ becomes arbitrarily close to a unique number L as x approaches c from either side, then the ________ of $f(x)$ as x approaches c is L.
2. To evaluate the limit of a polynomial function, use ________ ________.

Skills and Applications

3. Geometry You create an open box from a square piece of material 24 centimeters on a side. You cut equal squares with sides of length x from the corners and turn up the sides.

(a) Draw and label a diagram that represents the box.

(b) Write a function V that represents the volume of the box.

(c) The box has a maximum volume when $x = 4$. Use a graphing utility to complete the table and observe the behavior of the function as x approaches 4. Use the table to find $\lim_{x \to 4} V(x)$.

x	3	3.5	3.9	4	4.1	4.5	5
$V(x)$							

(d) Use the graphing utility to graph the volume function. Verify that the volume is maximum when $x = 4$.

4. Geometry A right triangle has a hypotenuse of $\sqrt{18}$ inches.

(a) Draw and label a diagram that shows the base x and height y of the triangle.

(b) Write a function A in terms of x that represents the area of the triangle.

(c) The triangle has a maximum area when $x = 3$ inches. Use a graphing utility to complete the table and observe the behavior of the function as x approaches 3. Use the table to find $\lim_{x \to 3} A(x)$.

x	2	2.5	2.9	3	3.1	3.5	4
$A(x)$							

(d) Use the graphing utility to graph the area function. Verify that the area is maximum when $x = 3$ inches.

Estimating a Limit Numerically In Exercises 5–10, complete the table and use the result to estimate the limit numerically. Determine whether it is possible to reach the limit.

5. $\lim_{x \to 1} (7x + 3)$

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$				?			

6. $\lim_{x \to -1} (3x^2 + 2x - 6)$

x	−1.1	−1.01	−1.001	−1
$f(x)$				?

x	−0.999	−0.99	−0.9
$f(x)$			

7. $\lim_{x \to -2} \dfrac{x + 2}{x^2 - 4}$

x	−2.1	−2.01	−2.001	−2
$f(x)$				?

x	−1.999	−1.99	−1.9
$f(x)$			

8. $\lim_{x \to 3} \dfrac{x - 3}{x^2 - 2x - 3}$

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$				?			

9. $\lim_{x \to 0} \dfrac{\sin 2x}{x}$

x	−0.1	−0.01	−0.001	0
$f(x)$				?

x	0.001	0.01	0.1
$f(x)$			

10. $\lim_{x \to 0} \frac{\tan x}{2x}$

x	-0.1	-0.01	-0.001	0
$f(x)$				?

x	0.001	0.01	0.1
$f(x)$			

Using a Graphing Utility to Estimate a Limit In Exercises 11–22, use a graphing utility to create a table of values for the function and estimate the limit numerically. Confirm your result graphically.

11. $\lim_{x \to 1} \frac{x - 1}{x^2 + 2x - 3}$

12. $\lim_{x \to -2} \frac{x + 2}{x^2 + 5x + 6}$

13. $\lim_{x \to 0} \frac{\sqrt{x + 5} - \sqrt{5}}{x}$

14. $\lim_{x \to -3} \frac{\sqrt{1 - x} - 2}{x + 3}$

15. $\lim_{x \to -4} \frac{\frac{x}{x + 2} - 2}{x + 4}$

16. $\lim_{x \to 2} \frac{\frac{1}{x + 2} - \frac{1}{4}}{x - 2}$

17. $\lim_{x \to 0} \frac{\sin^2 x}{x}$

18. $\lim_{x \to 0} \frac{2x}{\tan 4x}$

19. $\lim_{x \to 0} \frac{e^{2x} - 1}{2x}$

20. $\lim_{x \to 0} \frac{1 - e^{-4x}}{x}$

21. $\lim_{x \to 1} \frac{\ln(2x - 1)}{x - 1}$

22. $\lim_{x \to 1} \frac{\ln x^2}{x - 1}$

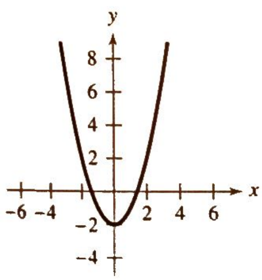
Using a Graph to Find a Limit In Exercises 23 and 24, graph the function and find the limit (if it exists) as x approaches 2.

23. $f(x) = \begin{cases} 2x + 1, & x < 2 \\ x + 3, & x \ge 2 \end{cases}$

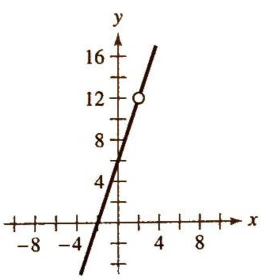
24. $f(x) = \begin{cases} -2x, & x \le 2 \\ x^2 - 4x + 1, & x > 2 \end{cases}$

Using a Graph to Find a Limit In Exercises 25–32, use the graph to find the limit, if it exists. If the limit does not exist, explain why.

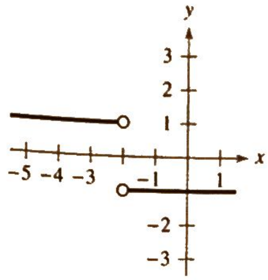
25. $\lim_{x \to -2} (x^2 - 2)$



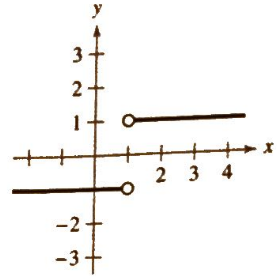
26. $\lim_{x \to 2} \frac{3x^2 - 12}{x - 2}$



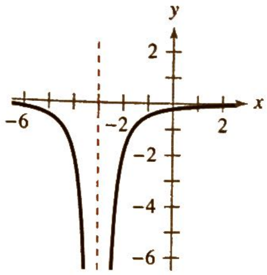
27. $\lim_{x \to -2} -\frac{|x + 2|}{x + 2}$



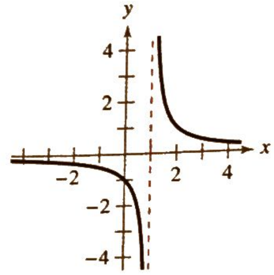
28. $\lim_{x \to 1} \frac{|x - 1|}{x - 1}$



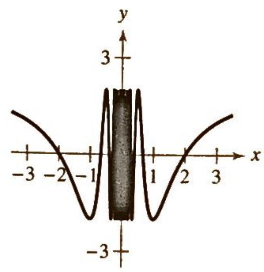
29. $\lim_{x \to -3} -\frac{2}{(x + 3)^2}$



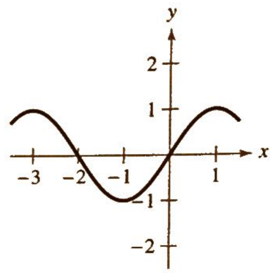
30. $\lim_{x \to 1} \frac{1}{x - 1}$



31. $\lim_{x \to 0} 2 \cos \frac{\pi}{x}$



32. $\lim_{x \to -1} \sin \frac{\pi x}{2}$



Determining Whether a Limit Exists In Exercises 33–40, use a graphing utility to graph the function and use the graph to determine whether the limit exists. If the limit does not exist, explain why.

33. $f(x) = \frac{5}{2 + e^{1/x}}, \quad \lim_{x \to 0} f(x)$

34. $f(x) = \ln(7 - x), \quad \lim_{x \to -1} f(x)$

35. $f(x) = \cos \frac{1}{x}, \quad \lim_{x \to 0} f(x)$

36. $f(x) = \sin \pi x, \quad \lim_{x \to 1} f(x)$

37. $f(x) = \frac{\sqrt{x + 3} - 1}{x - 4}, \quad \lim_{x \to 4} f(x)$

38. $f(x) = \frac{\sqrt{x + 5} - 4}{x - 2}, \quad \lim_{x \to 2} f(x)$

39. $f(x) = \frac{x - 1}{x^2 - 4x + 3}, \quad \lim_{x \to 1} f(x)$

40. $f(x) = \frac{7}{x - 3}, \quad \lim_{x \to 3} f(x)$

Evaluating Limits In Exercises 41 and 42, use the given information to evaluate each limit.

41. $\lim_{x\to c} f(x) = 3$, $\lim_{x\to c} g(x) = 6$

(a) $\lim_{x\to c} [-2g(x)]$ (b) $\lim_{x\to c} [f(x) + g(x)]$

(c) $\lim_{x\to c} \frac{f(x)}{g(x)}$ (d) $\lim_{x\to c} \sqrt{f(x)}$

42. $\lim_{x\to c} f(x) = 5$, $\lim_{x\to c} g(x) = -2$

(a) $\lim_{x\to c} [f(x) + g(x)]^2$ (b) $\lim_{x\to c} [6f(x)g(x)]$

(c) $\lim_{x\to c} \frac{5g(x)}{4f(x)}$ (d) $\lim_{x\to c} \frac{1}{\sqrt{f(x)}}$

Evaluating Limits In Exercises 43 and 44, find (a) $\lim_{x\to 2} f(x)$, (b) $\lim_{x\to 2} g(x)$, (c) $\lim_{x\to 2} [f(x)g(x)]$, and (d) $\lim_{x\to 2} [g(x) - f(x)]$.

43. $f(x) = x^3$, $g(x) = \frac{\sqrt{x^2+5}}{2x^2}$

44. $f(x) = \frac{x}{3-x}$, $g(x) = \sin \pi x$

Evaluating a Limit by Direct Substitution In Exercises 45–64, find the limit by direct substitution.

45. $\lim_{x\to 4} (8 - x^2)$

46. $\lim_{x\to -2} \left(\frac{1}{2}x^3 - 5x\right)$

47. $\lim_{x\to -3} (2x^2 + 4x + 1)$

48. $\lim_{x\to -3} (x^3 - 3x + 8)$

49. $\lim_{x\to 2} \left(-\frac{12}{x}\right)$

50. $\lim_{x\to -4} \frac{8}{x-4}$

51. $\lim_{x\to -2} \frac{2x}{2x^2-3}$

52. $\lim_{x\to 5} \frac{x-2}{x^2-3x+2}$

53. $\lim_{x\to -1} \frac{6x+5}{3x-7}$

54. $\lim_{x\to 3} \frac{x^2+1}{x}$

55. $\lim_{x\to -3} \sqrt{6-x}$

56. $\lim_{x\to 3} \sqrt[3]{x^2-1}$

57. $\lim_{x\to 7} \frac{5x}{\sqrt{x+2}}$

58. $\lim_{x\to 8} \frac{\sqrt{x+1}}{x-4}$

59. $\lim_{x\to 3} e^x$

60. $\lim_{x\to e} \ln x$

61. $\lim_{x\to \pi} \cos x$

62. $\lim_{x\to \pi/2} \tan 2x$

63. $\lim_{x\to 1/2} \arcsin x$

64. $\lim_{x\to 1} \arccos \frac{x}{2}$

Exploration

True or False? In Exercises 65 and 66, determine whether the statement is true or false. Justify your answer.

65. The limit of a function as x approaches c does not exist when the function approaches -3 from the left of c and 3 from the right of c.

66. The limit of the product of two functions is equal to the product of the limits of the two functions.

67. **Think About It** From Exercises 5–10, select a limit that is possible to reach and one that is not possible to reach.

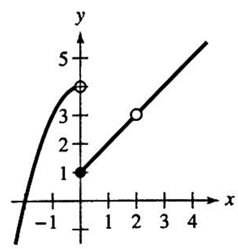
(a) Use a graphing utility to graph the corresponding functions using a standard viewing window. Do the graphs reveal whether it is possible to reach the limit? Explain.

(b) Use the graphing utility to graph the corresponding functions using a *decimal* setting. Do the graphs reveal whether it is possible to reach the limit? Explain.

68. **Think About It** Use the results of Exercise 67 to draw a conclusion as to whether you can use the graph generated by a graphing utility to determine reliably whether it is possible to reach a limit.

69. **Writing** Write a brief description of the meaning of the notation $\lim_{x\to 5} f(x) = 12$.

70. **HOW DO YOU SEE IT?** Use the graph of the function f to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.



(a) $f(0)$ (b) $\lim_{x\to 0} f(x)$ (c) $f(2)$ (d) $\lim_{x\to 2} f(x)$

Error Analysis In Exercises 71 and 72, describe the error.

71. When $f(2) = 4$, $\lim_{x\to 2} f(x) = 4$. ✗

72. When $\lim_{x\to 2} f(x) = 4$, $f(2) = 4$. ✗

73. **Think About It** Use a graphing utility to graph the tangent function. What are $\lim_{x\to 0} \tan x$ and $\lim_{x\to \pi/4} \tan x$? What can you conclude about the existence of the limit $\lim_{x\to \pi/2} \tan x$?

74. **Writing** Use a graphing utility to graph the function

$$f(x) = \frac{x^2 - 3x - 10}{x - 5}.$$

Use the *trace* feature to approximate $\lim_{x\to 4} f(x)$. What appears to be the value of $\lim_{x\to 5} f(x)$? Is f defined at $x = 5$? Does this affect the existence of the limit as x approaches 5?